Question 9 (6 marks)

Particles and are moving with constant velocities and have initial positions m and m respectively. seconds later is at m.

(a) Determine the velocity of . (1 mark)

The velocity of is m/s.

(b) Show that the paths of and cross, state the position vector of this point, and explain whether the particles collide. (5 marks)

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(a) Determine the velocity of . (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct velocity |

The velocity of is m/s.

(b) Show that the paths of and cross, state the position vector of this point, and explain whether the particles collide. (5 marks)

|  |
| --- |
| Solution |
| For paths to cross we require . Equating and coefficients and solving simultaneously:  Check coefficients are equal with these values of and :  Because , their paths cross at this point and because both particles reach this point at the same time they collide. |
| Specific behaviours |
| ✓ indicates equations for both paths  ü forms two equations using different time parameters  ü solves equations and checks third coefficient  ü correct position vector  ü explains why paths cross and whether particles collide |

Question 15 (6 marks)

<EFOFEX>
id:fxd{fda65e44-d5bd-4637-8cca-3d8ffd208507}

FXData:

</EFOFEX>The diagram shows a right rectangular prism.

Relative to vertex , vertices and have  
position vectors and .

(a) Determine vectors and in terms of and . (1 mark)

(b) Use a vector method to show that diagonals and bisect each other. (3 marks)

(c) Determine the relationship between and when and are perpendicular.

(2 marks)

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(a) Determine vectors and in terms of and . (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct vectors |

(b) Use a vector method to show that diagonals and bisect each other. (3 marks)

|  |
| --- |
| Solution |
| Midpoint of line :  Midpoint of line :  Since then diagonals are coincident at their midpoints and so bisect each other. |
| Specific behaviours |
| ✓ develops expression for position vector of one midpoint  ü develops expression for position vector of one midpoint  ü shows midpoints are coincident and hence bisect |

(c) Determine the relationship between and when and are perpendicular.

(2 marks)

|  |
| --- |
| Solution |
| For vectors to be perpendicular, require .  Hence or . |
| Specific behaviours |
| ü indicates condition for perpendicularity  ü correct relationship |

Question 17 (7 marks)

Plane has equation and point has coordinates .

(a) Determine the coordinates of the point in that is closest to . (4 marks)

Vector lies in plane , is perpendicular to the line and .

(b) Let . Determine the value of coefficients and , given that .

(3 marks)

Question 17 (7 marks)

Plane has equation and point has coordinates .

(a) Determine the coordinates of the point in that is closest to . (4 marks)

|  |
| --- |
| Solution |
| Equation of line through is .  Intersects with plane when  Coordinates are . |
| Specific behaviours |
| ✓ equation of line through point  ü uses intersection to obtain equation in  ü solves for  ü correct coordinates |

Vector lies in plane , is perpendicular to the line and .

(b) Let . Determine the value of coefficients and , given that .

(3 marks)

|  |
| --- |
| Solution |
| Solving equations simultaneously: |
| Specific behaviours |
| ✓ two equations using normals  ü equation using magnitude  ü correct set of values |

Question 8 (7 marks)

Point lies on a sphere with centre , radius and diameter .

(a) Let and . Use a vector method to prove that is perpendicular to .

(4 marks)

(b) If the position vectors of and are and respectively, determine the value of the constant . (3 marks)

Question 8 (7 marks)

Point lies on a sphere with centre , radius and diameter .

(a) Let and . Use a vector method to prove that is perpendicular to .

(4 marks)

|  |
| --- |
| Solution |
| But points on sphere so that.  Hence and since and we deduce that the angle between and must be . |
| Specific behaviours |
| ✓ correct vectors for and  ü forms and expands scalar product  ü uses explains that  ü deduces perpendicularity |

(b) If the position vectors of and are and respectively, determine the value of the constant . (3 marks)

|  |
| --- |
| Solution |
| Hence . |
| Specific behaviours |
| ✓ vectors for and  ü calculates scalar product  ü correct value of |

Question 11 (7 marks)

A small body is moving with constant velocity in space so that initially it is located at and four seconds later it is at , where all dimensions are in metres.

(a) Determine a vector equation for the position of the small body at time seconds.

(2 marks)

A laser beam shines along the line with equation

(b) Write the vector equation of this line in the form . (1 mark)

(c) Show that the small body passes through the laser beam and state where this occurs.

(4 marks)

Question 11 (7 marks)

A small body is moving with constant velocity in space so that initially it is located at and four seconds later it is at , where all dimensions are in metres.

(a) Determine a vector equation for the position of the small body at time seconds.

(2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ calculates velocity vector  ü correct equation for position of body |

A laser beam shines along the line with equation

(b) Write the vector equation of this line in the form . (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct vector form |

(c) Show that the small body passes through the laser beam and state where this occurs.

(4 marks)

|  |
| --- |
| Solution |
| Equating and coefficients:  Solving simultaneously gives .  Using these values, the body is at and the laser passes through .  Hence as these points are coincident, the small body passes through the laser beam at this point. |
| Specific behaviours |
| ✓ equates two coefficients  ü solves simultaneously  ü calculates both coefficients or points and states they are same  ü states point of coincidence |

Question 13 (6 marks)

Points , and lie in plane with position vectors and respectively.

(a) Determine the vector equation for plane in the form . (3 marks)

The equation of line is .

(b) Determine, if possible, where line intersects with plane . If not possible, explain why not. (3 marks)

Question 13 (6 marks)

Points , and lie in plane with position vectors and respectively.

(a) Determine the vector equation for plane in the form . (3 marks)

|  |
| --- |
| Solution |
| Hence equation of is |
| Specific behaviours |
| ✓ obtains two vectors in the plane  ü obtains normal to plane  ü obtains value of and states equation of plane |

The equation of line is .

(b) Determine, if possible, where line intersects with plane . If not possible, explain why not. (3 marks)

|  |
| --- |
| Solution |
| Substitute equation of line into equation of plane:  Hence  Since the equation is true and the solution of the equation is independent of then all values of are solutions. Hence line lies in plane and all points on are points of intersection. |
| Specific behaviours |
| ✓ substitutes equation of line into equation of plane  ü simplifies  ü reasons that all points on the line lie in the plane |





























